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Compressibility effect on the gas flow and heat transfer in a microtube

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1. INTRODUCTION

They fluid flow and heat transfer in microtubes has become a subject of recent investigations. This is because microchannels with dimensions ranging form 100 micrometer to fractions of 1 micrometer have found applications in such areas as microelectronics, biological cell reactors and micro heat exchangers, etc. The existing experimental results for microtubes indicated significant departure from the flow and heat transfer correlations used for conventional sized tubes. However, the experimental data reported by different authors quantitatively and even qualitatively conflicted with each other in some cases. Wu and Little [1] measured the friction factors for the flow of gases in very fine channels used for microminiature Joule–Thomson refrigerators. The width of the channels was 130–200 microns and the depth 30–60 microns. The tests involved both laminar and turbulent flow regimes. The results obtained showed differing features than those expected in traditional flows. Friction factor correlations were presented as a function of the Reynolds number. The product ($f\,Re$) can be up to 118 depending on surface roughness in the channels, which is much larger than the value of 64 for the laminar flow in conventional channels. In addition, the flow transition from laminar to turbulent occurred at Reynolds number of about 1000. Pfahler *et al.* [2, 3] conducted a sequence of experimental studies with gases and liquids to measure the friction factor in microchannels ranging in size from 0.5 to 50 microns. The flow was assumed to be fully developed since the ratio of the channel length to the hydraulic diameter is very large. They were not able to find calculations of the friction factor for compressible gas flows and cited the observations from Keenan and Neumann [4] and Shapiro [5] that the incompressible value was applicable to some subsonic compressible flows. A difference was found between the measured friction factor and the one predicted by incompressible theory. The friction factors were consistently lower than the theoretical predictions. Furthermore, the friction factor decreased with decreasing Reynolds number for laminar flows at small Reynolds number. Wu and Little [6] measured the heat transfer coefficients of nitrogen gas through four microchannels test components over a channel height range from 89 to 97 microns and a channel width range from 312 to 572 microns. Experimental results exhibited laminar, transition and turbulent heat transfer regimes that were separated by the Reynolds number of 1000 and 3000. The Nusselt number varied with the Reynolds number for the laminar flow and it is difficult to correlate the heat transfer data for the transitional regime. The Reynolds analogy was not valid for the turbulent flow in the rough channels tested. Choi *et al.* [7] measured the friction factors and convective heat transfer coefficients for flow of nitrogen gas in microtubes for both laminar and turbulent flow regimes. The microtube inside diameters ranged from 3 to 81 microns. The experimental results indicated significant departure from the thermofluid correlations used for conventional-sized tubes. For microtube diameter of smaller than 10 microns or for Reynolds number of smaller than about 400, the friction factor correlation, $C = f\,Re$, produced a value for the constant $C = 53$, instead of 64. The measured Nusselt numbers for laminar heat transfer exhibited a strong function of Reynolds number and for turbulent heat transfer in microtubes were as much as 7 times the values predicted by the Colburn analogy, $j = f/8$. Duncan and Peterson [8] presented a review of literature in the area of microscale heat transfer, including the microscale forced convection, to provide a concise overview of the recent advances in this field of study.

There may be several factors making the gas flow and heat transfer behavior different in microtubes from that in normal-sized tubes. They can be largely summarized in: (1) the gas compressibility. The compressibility induced flow acceleration has been taken into consideration in evaluating the pressure drop in micro channels [1, 3, 7], but, the compressibility effect on the friction factor was believed to be negligible for subsonic flows [1–7]; (2) the surface roughness. The surface relative roughness had great influence on the flow and heat transfer for laminar flows as well as turbulent flows in microchannels [1, 7]; (3) the gas rarefaction. The degree of rarefaction is measured by the Knudsen number which is the ratio of the mean free path of gas to the characteristic dimension of the flow structure. The effect of rarefaction will lower the apparent viscosity and consequent friction factor as the Knudsen number larger than 0.01 [3]. However, there was no satisfactory explanation for the experimental data obtained in Ref. [7].

In view of the fact that no analytical or computational work is available about the effect of flow compressibility on the flow and heat transfer behavior, the objective of this paper is to numerically examine the compressibility effect on the friction coefficient and Nusselt number of gas flows in smooth microtubes.

2. GOVERNING EQUATIONS AND NUMERICAL PROCEDURE

The governing equations for the compressible flow and heat transfer in a circular tube are those of continuity, momentum, energy and state. These equations expressed in cylindrical coordinates and in accordance with Fig. 1, take the following forms

Continuity:

$$\frac{\partial}{\partial x}(ru) + \frac{1}{r}\frac{\partial}{\partial r}(rv) = 0. \quad (1)$$

Axial momentum:

$$\frac{\partial}{\partial x}(\rho uu) + \frac{1}{r}\frac{\partial}{\partial r}(\rho rvu) = -\frac{\partial p}{\partial x}$$

NOMENCLATURE

C_f	friction coefficient $C_f = \tau_w/(\frac{1}{2})\rho u_m$	λ	thermal conductivity
C_p	specific heat	μ	dynamic viscosity
k	ratio of specific heat	ρ	density
M	Mach number	θ	dimensionless temperature.
p	pressure		
Pr	Prandtl number		
Re	Reynolds number		
u	axial velocity		
v	radial velocity.	Subscripts	
		0	inlet section
Greek symbols		m	mean
α	heat transfer coefficient	w	wall
	$\alpha = q_w/T_w - T_m$	f	friction.

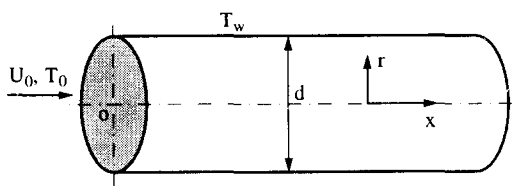

Fig. 1. Schematic of the tube flow and the coordinates.

$$+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right). \quad (2a)$$

Radial momentum:

$$\frac{\partial}{\partial x}(\rho uv)+\frac{1}{r}\frac{\partial}{\partial r}(\rho rvv) = -\frac{\partial p}{\partial r}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial v}{\partial r}\right)-\frac{v}{r^2}. \quad (2b)$$

Energy:

$$\frac{\partial}{\partial x}(\rho C_p uT)+\frac{1}{r}\frac{\partial}{\partial r}(\rho C_p rvT) = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right). \quad (3)$$

State:

$$p = \rho RT. \quad (4)$$

Subject to the boundary conditions:

$$x = 0: \quad u = 2[1-4(r/d)^2]u_m, v = 0, T = T_0,$$
$$x > 0, r = 0.5d: \quad u = v = 0, T = T_w, \quad (5)$$

where the axial diffusion terms in the above equations have been dropped, since the transport of momentum and energy in the axial direction is dominated by the convection of main flow. The work and dissipation terms are excluded in the energy equation (3) due to their less importance than other terms for the subsonic flows. All variables, the governing equations and boundary conditions are nondimensionalized by introducing the following characteristic parameters: the diameter of the tube, d; the inlet average velocity, u_{m0}; the inlet gas density, ρ_0; the inlet dynamics pressure, $\rho_0 u_{m0}^2$, and the inlet gas temperature T_0 and the wall temperature T_w, respectively. The new variables are

$$X = \frac{x}{d}, \quad R = \frac{r}{d}, \rho^* = \frac{\rho}{\rho_0}; \quad P = \frac{p}{\rho_0 u_{m0}^2};$$

$$U, V = \frac{u, v}{u_{m0}}, \quad \theta = \frac{T-T_0}{T_w-T_0} \quad Re = \frac{\rho_0 d u_{m0}}{\mu}; \quad M = \frac{u_m}{\sqrt{kRT}}$$

and the nondimensionalized governing equations are

$$\frac{\partial}{\partial X}(\rho^* U)+\frac{1}{R}\frac{\partial}{\partial R}(\rho^* RV) = 0 \quad (6)$$

$$\frac{\partial}{\partial X}(\rho^* UU)+\frac{1}{R}\frac{\partial}{\partial R}(\rho^* RVU) = -\frac{\partial P}{\partial X}+\frac{\partial}{\partial X}\left(\frac{1}{Re}\frac{\partial U}{\partial X}\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{1}{Re}R\frac{\partial U}{\partial R}\right) \quad (7a)$$

$$\frac{\partial}{\partial X}(\rho^* UV)+\frac{1}{R}\frac{\partial}{\partial R}(\rho^* RVV) = -\frac{\partial P}{\partial R}+\frac{\partial}{\partial X}\left(\frac{1}{Re}\frac{\partial V}{\partial X}\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{1}{Re}R\frac{\partial V}{\partial R}\right)-\frac{1}{Re}\frac{V}{R^2} \quad (7b)$$

$$\frac{\partial}{\partial X}(\rho^* U\theta)+\frac{1}{R}\frac{\partial}{\partial R}(\rho^* RV\theta) = -\frac{\partial P}{\partial X}+\frac{\partial}{\partial X}\left(\frac{1}{Re\,Pr}\frac{\partial \theta}{\partial X}\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{1}{Re\,Pr}R\frac{\partial \theta}{\partial R}\right) \quad (8)$$

$$P = \frac{\rho^*}{kM_0^2}\frac{T}{T_0} \quad (9)$$

subject to boundary conditions

$$X = 0: \quad U = 2(1-4R^2), V = 0, \theta = 0;$$
$$X > 0, R = 0.5: \quad U = V = 0, \theta = 1. \quad (10)$$

The fluid properties except the density were assumed to be constant during solving equations so that the flow problem can be solved without reference to the energy equation. In addition, the flow was assumed to be approximately isothermal for predicting the friction coefficient, since there was sufficient heat transfer area for a microtube with large length–diameter ratio. The obtained velocity distribution was put into energy equation, which was solved for the temperature distribution and the Nusselt number. In view of the fact that the density is dependent on the pressure only for the isothermal flow and largely on the pressure for the compressible flow with small temperature difference, the equation of state (9) can be reduced in

$$\rho^* = \frac{\rho}{\rho_0} = \frac{p}{p_0} = PkM_{m0}^2 = 1-(P_0-P)kM_{m0}^2. \quad (11)$$

The uncoupling of axial and lateral pressure gradient technique was adopted for the parabolic flow, and the governing equations were solved by means of a finite difference forward-matching procedure [9]. The upwind scheme was used for axial direction and power law for the lateral direction [10]. A uniform grid 50 (radial) × 400 (axial) was set and the computation were terminated when the residuals for all parameters less than 10^{-4}.

3. RESULTS AND DISCUSSIONS

3.1. *Pressure and density variations*

Based on one-dimensional analysis [5], the local pressure variations for the isothermal flow in a constant area tube is given

$$\frac{dp}{p} = -\frac{kM^2}{2(1-kM^2)}4C_f\frac{dx}{d} = -128\frac{kM^2}{(1-kM^2)}\frac{dx}{Re\,d}. \quad (12)$$

It can be found that the pressure drop between the tube inlet and outlet is usually much smaller than the local pressure itself for the gas flow in a conventional scaled tube, since both the Mach number and $(l/d)/Re$ are much smaller than unity. Thus the density variation of gas along the tube is negligible and the flow can then be assumed to be incompressible with good accuracy. However, the gas flow in a microtube commonly has a large value of l/d and small value of Re, which result in the friction produced pressure drop comparable to the local pressure and in the considerable density variation along the tube. As a result, the pressure drop induced by gas acceleration in the flow direction has to be taken into consideration. The ratio of the friction pressure drop to the acceleration pressure drop can be evaluated by

$$\frac{dp_f}{dp_a} = \frac{1-M^2}{kM^2} \quad (13)$$

which is dependent on the Mach number only. Should the Mach number of gas flow exceed 0.2, the acceleration pressure drop cannot be neglected.

3.2. *Velocity profile and friction coefficient*

For the fully developed flow it is well-known that the axial velocity gradient $\partial u/\partial x = 0$, that the radial velocity component $v = 0$ and that the pressure gradient keeps constant. Consequently, the velocity profile is parabolic and the friction factor $f = 64/Re$ or the friction coefficient $C_f = 16/Re$ for the steady flow in a circular tube. For compressible tube flow, the gas acceleration leads to the velocity profile change not only in amount, but also in shape. The former produces the additional pressure drop, while the latter gives rise to the departure of the friction factor from its conventional correlation. This is similar to the fact that the presence of the pressure gradient in the flow boundary layer will affect the shape of the velocity profile and the skin-friction coefficient. When the compressibility effect of gas flow on the velocity profile shape must be accounted for, the one-dimensional study is inadequate. Therefore, numerical solutions of two-dimensional governing equations (6)–(10) describing the compressible laminar flows in a circular smooth tube was carried out. Figure 2 shows the dimensionless velocity profile, which greatly differs from the parabolic velocity profile, at the position of $(l/d)/Re = 0.985$ for the gas flow with $M_0 = 0.1$. Furthermore, it is clearly seen in Fig. 2 that the gas acceleration brings about the increase of the dimensions velocity gradient at the wall surface. Figure 3 demonstrates the dependence of the friction coefficient on

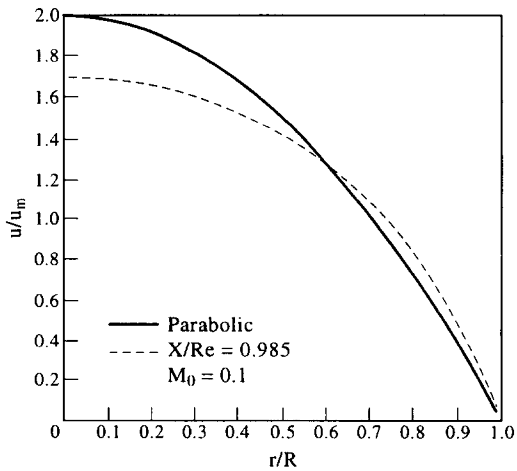

Fig. 2. Compressibility effect on the velocity profile: — velocity profile at inlet; --- velocity profile at $X/Re = 0.985$.

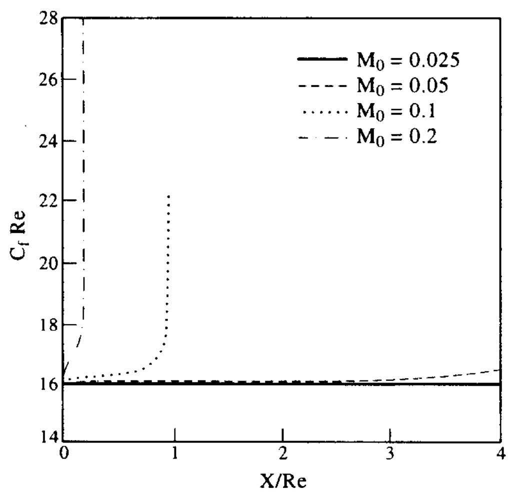

Fig. 3. Dependence of $C_f\,Re$ on X/Re with the inlet Mach number as a parameter.

the normalized tube length with the inlet Mach number as a parameter. The friction coefficient increases with increasing $(l/d)/Re$ resulting from the increase of the dimensionless velocity gradient at the wall surface. It is worth noting that the fully developed velocity profile can never be established for a compressible tube flow even if the tube is very long or the normalized tube length is very large. In addition, the inlet Mach number, M_0, has great impact to the friction coefficient. This is because the Mach number itself represents the magnitude of compressibility of gas flow and, thus, the higher the Mach number is, the more the velocity profile deviates from the parabolic one and the larger the friction coefficient becomes, as shown in Fig. 3.

3.3. *Temperature profile and Nusselt number*

Knowing that the temperature profile is strongly dependent on the velocity profile of flow in a tube, it is therefore evident that no fully developed temperature profile can be reached as long as the flow is not yet fully developed. This implies that the fluid flow can never be fully developed both dynamically and thermally for a compressible tube flow. It turns out that the local Nusselt number, like the local friction

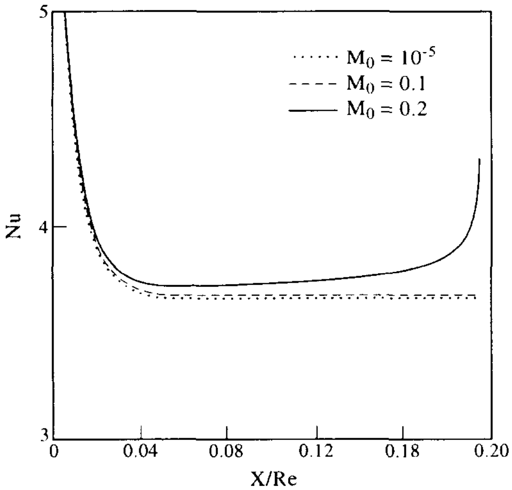

Fig. 4. Dependence of Nu on X/Re with the inlet Mach number as a parameter.

coefficient, increases along the flow direction, rather than keeps constant. The solutions of the energy equation (8) subjected to the constant wall temperature and the uniform gas temperature at the tube inlet are illustrated in Fig. 4. When the Mach number is much smaller than unity, the compressibility effect of gas flow is so weak, that the Nusselt number sharply falls down in the inlet region and approaches a limit of 3.65 for the fully developed region at $X/Re = 0.04$. These agree well with the incompressible results. For the flow in a microtube characterized with large values of the normalized tube length and Mach number, however, the local Nusselt number is no longer constant, but increases along the tube due to the change in dimensional velocity gradient and consequent dimensional temperature gradient at the wall surface.

4. CONCLUDING REMARKS

(1) The friction induced pressure gradient and the consequent flow acceleration for gas flows in microtubes can be so large, that they result in the increase of the dimensionless velocity gradient at the wall surface, and the consequent increase of the local friction coefficient compared with that for conventional sized tubes.

(2) The fully developed gas flow in microtubes is unavailable even at very large values of $(l/d)/Re$ because the radial profile of the axial velocity changes continuously along the tube due to the compressibility effect. The product of the friction coefficient and Reynolds number, $C_f\,Re$, is therefore no longer constant, but dependent on Reynolds number.

(3) The local Nusselt number increases with increasing the dimensionless length due to the compressibility effect, instead of being constant for gas flows in normal scaled tubes. That is, the thermally fully developed flow can never be established for a compressible tube flow because of the strong dependence of the temperature profile on the velocity profile.

(4) The tube inlet Mach number has great impact on both the friction coefficient and Nusselt number. The higher the inlet Mach number is, the larger the friction coefficient and the Nusselt number become.

(5) The neglect of the flow compressibility effect may be one of the reasons for the discrepancy between experimental results of flow and heat transfer in microtubes from different authors.

REFERENCES

1. Wu, P. Y. and Little, W. A., Measurement of friction factors for the flow and gases in very fine channels used for microminiature Joule–Thomson refrigerators. *Cryogenics*, 1983, **23**, 273–277.
2. Pfahler, J. N., Harley, J., Bau, H. and Zemel, J., Liquid and gas transport in small channels. *ASME DSC*, 1990, **19**, 149–157.
3. Pfahler, J. N., Harley, J., Bau, H. and Zemel, J., Gas and liquid flow in small channels. *ASME DSC*, 1990, **32**, 49–60.
4. Keenan, J. H. and Neumann, E. P., Measurement of friction in a pipe for subsonic and supersonic flow of air. *Journal of Applied Mechanics*, 1946, **13**, 91–102.
5. Shapiro, A. H., *The Dynamics and Thermodynamics of compressible Fluid Flow*. The Ronald Press Company, New York, 1953.
6. Wu, P. Y. and Little, W. A., Measurement of the heat transfer characteristics of gas flow in fine channel heat exchanger used for microminiature refrigerators. *Cryogenics*, 1984, **24**, 415–420.
7. Choi, S. B., Barrori, R. G. and Warrington, R. W., Fluid flow and heat transfer in microtubes. *ASME DSC*, 1991, **32**, 123–134.
8. Duncan, A. B. and Peterson, G. P., Review of microscale heat transfer. *Applied Mechanics Review*, 1994, **46**, 397–428.
9. Patankar, S. V. and Spalding, D. B., A calculation procedure for heat mass and momentum transfer in three-dimensional parabolic flow. *International Journal of Heat and Mass Transfer*, 1972, **15**, 1787–1806.
10. Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*. Hemisphere Publishing Company, New York, 1980.